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Fractional Calculus: A New Look

A.M. Mathai
Director, Centre for Mathematical and Statistical Sciences
Peechi Campus, KFRI, Peechi-680653, Kerala, India
directorcms458@gmail.com
and
Emeritus Professor of Mathematics and Statistics
McGill University, Montreal, Canada, H3A 2K6
mathai@math.mcgill.ca

Abstract

In this article we give an overview of the recent developments in the area of fractional integrals and fractional derivatives. A new definition is given by this author in terms of Mellin convolutions of ratios and products in the case of real scalar variables and M-convolutions of ratios and products in the case of matrix variables, where one of the functions is a type-1 beta type so that all the definitions available in the literature for fractional integrals can be brought under one definition. Once the fractional integrals are defined, fractional derivatives can be defined as certain fractional integrals so that the results coming from fractional derivatives can describe global activities compared to integer order derivatives which can describe only local activities at a point. When fractional derivatives are defined as certain fractional integrals then these derivatives cover not only given points of interest but also their neighborhoods so that fractional derivatives become more useful in practical applications. An ideal situation may be a local activity but in reality the real-life situation may be in the neighborhood of the ideal case. The new definition is also extended to real matrix-variate case as well as to complex matrix-variate case. Thus, for the first time, fractional calculus of functions of complex variables is also given through the new definition.

Keywords: Fractional integrals, fractional derivatives, scalar and matrix variate cases, complex-variate cases, Mellin convolutions, M-convolutions of products and ratios.

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1. Introduction.

Even though fractional calculus is as old as integer order calculus itself, the area of fractional calculus was dormant all these years except for the last two decades.